Exam I Sample Questions, MA1025, Winter 2004
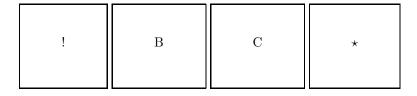
The following seven problems were taken from a couple of old first exams. Typically, an exam will feature four problems (possibly with parts).

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1. Is $\{\emptyset\} = \emptyset$? Explain.

2. Assume that the universal set is the set of real numbers. Prove that the propositions "$\exists x(P(x) \wedge Q(x))$" and "$\exists x P(x) \wedge \exists x Q(x)$" are in general not equivalent, by producing statements P and Q for which the propositions in question have opposite truth values.

3. Claim: For all real numbers t, if t is irrational then $5t$ is irrational.

   "Proof:" Let $t \in \mathbf{R}$, and suppose $5t$ is rational. Then $5t = a/b$, where $a, b \in \mathbf{Z}$ and $b \neq 0$. But then $t = a/5b$, so t is rational. We conclude that if t is irrational then $5t$ is irrational.   □

   Describe the proof in logical terms, using symbols. Is the proof correct?

4. Following are descriptions of two sets using the "set-generating" notation. Describe each set by listing its elements between braces.

   (a) $\{x \in \mathbf{Z} | -8 \le x^3 \le 8\}$
   (b) $\{x \in \mathbf{N} | \exists y \in \mathbf{N}(x^2 + y^2 \le 50)\}$

5. Assume that the universal set is the set of real numbers. Decide whether each of the following is true or false, and in each case explain your reasoning.

   (a) $\forall x \exists! y (xy = y)$
   (b) $\exists! x \forall y (xy = y)$

6. Suppose we are shown four cards that are lying on a table. We know that one side of each card is labeled either A, B, or C, and that the other side of each card is labeled either !, $\star$, or Δ. The cards appear as follows:

   We are told that if any card bears an A or a B on one side then it must bear a Δ or a ! on the other side. Precisely which cards need to be turned over to either verify or refute this claim? Explain.

7. Describe each set using the form $\{f(x)|P(x)\}$:

   (a) $A = \{2, 6, 10, 14, ...\}$
   (b) $B = \{1, 1/2, 1/4, 1/8, ...\}$
   (c) $C = \{0, 7, 26, 63, 124, ...\}$